Research on Mathematical Teaching Strategies Based on HPM and CAI Theories

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Abstract: Based on the theories of HPM (History and Pedagogy of Mathematics) CAI (Computer Aided Instruction), this research deeply discusses the mathematics teaching strategies based on these two theoretical frameworks. Under the guidance of occurrence teaching method, cognitive load theory and multimedia learning theory, this paper puts forward the junior middle school mathematics teaching strategy based on HPM and CAI theory, it is put forward that interactive software, virtual technology and adventure games can be used to assist teaching while multimedia is used in class.

1. Introduction

1.1 Research Background of HPM Theory

At the second International Congress on Mathematical Education held in 1972, the International Study Group on the Relations Between the History and Pedagogy of Mathematics (commonly known as HPM) was established. Renowned mathematician Freudenthal articulated his HPM philosophy during the conference, stating, "If one regards mankind as a learner, then the history of mathematics is his process of learning mathematics," and "Children's learning should follow the reconstructed history" [1]. In 2005, China hosted the first National Conference on the History of Mathematics and Mathematics Education. Since then, more scholars have turned their attention to and engaged in research on HPM [2]. Understanding the historical development of mathematical curricula can support its teaching by demonstrating the roots of modern mathematics in the past and revealing the improvement of mathematical rigor [3].

1.2 Research Background And Significance of CAI Theory

With the rise of computer technology and the demand for reforms in mathematical education, the utilization of computer technology and related software has been employed to enhance the effectiveness of mathematics teaching and improve the learning experience [4]. In the field of mathematical education, CAI theory can be applied to various aspects, including the visualization of mathematical concepts, solving and exploring mathematical problems, and the establishment and analysis of mathematical models. Additionally, the quality and effectiveness of educational resources are crucial issues, necessitating the assurance that the design and development of instructional software and platforms align with educational objectives.

1.3 Current Situation of Research on HPM Education Concept

Internationally, research on HPM has a longer history, and the achievements are relatively abundant, especially with active engagement from mathematics educators in the United States. In mainland China, research on HPM by mathematicians started later, beginning in the early 21st century. In recent years, there has been significant development, notably demonstrated by the regular occurrence of the National Symposium on the History of Mathematics and Mathematics Education every two years (initiated in 2005). The incorporation of the history of mathematics into the "Ordinary High School Mathematics Curriculum Standards (Experimental)" has marked a milestone, and a considerable number of outstanding HPM research experts have emerged in this field in China. Notable figures include Professor Wang Xiaoqin from East China Normal University, Professor

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Zhang Weizhong from Zhejiang Normal University, and Professor Huang Youchu from Wenzhou University [5].

1.4 Current Situation of Research on Computer-Aided Mathematics Education

In 2020, Dong Xiaochen, Liu Zuhong, Wang Hongfei, and other faculty members from the School of Computer Science and Technology at Guizhou University believed that the integration of computer networking technology with traditional methods of mathematics instruction not only reduced the workload for teachers but also cultivated a strong interest in learning among students [6].

In 2021, Teacher Kang Lili asserted that computer-assisted teaching in middle school mathematics is advantageous for driving reforms in mathematical instruction, innovating teaching methodologies, and empowering student-centered learning. However, computer-assisted teaching in middle school mathematics also faces challenges such as lack of innovative approaches, relatively fewer platforms, an incomplete system, and a deficiency in integrated methods [7].

2. Concept Definition and Theoretical Basis

2.1 Introduction of related Concepts

2.1.1 The theoretical connotation of HPM

HPM (History and Pedagogy of Mathematics) is a crucial field within mathematics education. It is primarily employed to investigate why and how the history of mathematics is integrated into mathematical education. Scholars have embarked on a series of studies exploring these questions. In the realm of mathematical instruction, the history of mathematics serves as a robust assurance and support during the teaching process. Through the exploration of mathematical history, it unveils educational patterns, transforms traditional mathematical knowledge into forms conducive to student comprehension, thereby fostering students' learning of mathematics.

2.1.2 The concept and characteristics of CAI

Computer Aided Instruction (CAI) refers to an educational method that utilizes computer technology to enhance, improve, or support the process of education and learning. It enables personalized learning experiences and encourages active student engagement in the learning process. CAI systems typically incorporate learning analytics capabilities, allowing for the tracking of students' progress and performance. Additionally, CAI provides a wide array of rich educational resources and has the capacity to transcend geographical and cultural boundaries, making educational resources accessible across different regions and cultures.

2.2 Related Theoretical Basis

2.2.1 Teaching method of occurrence

The understanding of historical similarities by mathematics educators has led to the emergence of the "problem-based learning" approach. The foundation of this teaching method lies in the history of mathematics, and its fundamental principle requires an appropriate timing in students' psychological development, generating sufficient learning motivation behind the wheels of the subject.

This enables teachers to better apply mathematical knowledge to analyze and solve real-world problems while also enhancing their innovative and collaborative skills. Problem-based learning is also an instructional approach based on constructivist learning theory, emphasizing that learners construct and develop knowledge through personal experiences and reflection.

2.2.2 Cognitive load theory

When Cognitive Load Theory, proposed by educational psychologists such as John Sweller, aims to delve into the phenomenon of cognitive load during human learning and thinking. The goal of Cognitive Load Theory research is to minimize the cognitive load generated by learners during the learning process [8], providing learners with a more effective learning environment to address individual differences among learners. Interactivity and immediate feedback are additional

advantages of computer-aided instruction. It can also offer virtual experiments, simulated scenarios, and interactive exercises.

2.2.3 Theory of multimedia learning

The Multimedia Learning Theory, based on principles of human cognition and learning, seeks to investigate how multimedia technology can most effectively facilitate the process of learning and knowledge transfer. When combined with computer-aided instruction, the Multimedia Learning Theory provides crucial guidance and principles for the educational field to enhance the design and implementation of education.

3. Research on Mathematical Teaching Strategies Based on HPM and CAI Theories

3.1 Make Use of Multimedia Resources to Introduce Interesting Class

Utilizing multimedia means for digital presentations, showcasing significant events and figures in the history of mathematics [9]. Employing high-quality images, charts, and video materials, whether through slideshows, electronic whiteboards, or online teaching platforms, to vividly illustrate the evolution of mathematical theories and contributions of mathematicians.

Animating and simulating abstract mathematical concepts in a lively and vivid manner for students [10]. Through animation, demonstrating the proof processes of mathematical theorems, emphasizing the logic and steps involved in mathematical reasoning. Simulation software can simulate the problem-solving approaches of mathematicians, immersing students in practical mathematical practices.

Establishing virtual museum resources to provide students with opportunities to explore mathematics-related exhibitions worldwide. Teachers can guide students in browsing mathematical historical artifacts, manuscripts, and tools exhibited in the museum, enabling a deeper understanding of the evolutionary process of mathematics.

3.2 Based on Interactive Software, Heuristic Mathematics Learning is Realized

Designing an interactive learning software "Math Story" based on the history of mathematics, employing interactive and entertaining elements to assist students in exploring crucial events, mathematical concepts, and contributions of mathematicians. The software encompasses the following key features:

1) Interactive Mathematical History Overview: Provides an interactive overview of mathematical history, including a timeline, significant events, and biographies of mathematicians. Students can explore the development and milestones of mathematical history through this guided tour.

2) Multimedia Explanation of Key Concepts: Utilizes multimedia elements such as graphics, animations, and videos to explain important concepts in mathematical history, such as geometry, algebra, number theory, etc. Clear and concise language accompanied by visual examples aids students in comprehending the significance of these concepts in the history of mathematics.

3) Mathematician Profiles: Offers in-depth profiles of various mathematicians, detailing their backgrounds, contributions, and significant achievements. Through vivid storytelling and relevant images, students gain a better understanding of the work and impact of mathematicians.

4) Interactive Learning Activities: Designs a variety of interactive learning activities, including fill-in-the-blanks, multiple-choice questions, puzzles, and riddles, engaging students in reinforcing their understanding and memory of mathematical history. Each activity provides immediate feedback and solutions to assist students in mastering and digesting the learned content.

5) Regular Quizzes and Assessments: Provides periodic quizzes and assessments to help students evaluate their understanding and mastery of mathematical history. Quiz results are compiled to generate individual learning reports, offering personalized learning recommendations and further learning resources.

6) User Community: Establishes a user community where individuals can share learning experiences, discuss topics related to the history of mathematics, and interact with other users.

Facilitates interaction with experts and educators in the field of mathematical history, providing professional guidance and clarification.

Incorporating these features, "Math Story" aims to create an immersive and effective learning environment that not only imparts knowledge but also fosters engagement and collaboration within a community of learners.

3.3 Create a Simulation Scene, Really Experience the Development of Mathematics

Design an application called "Math VR" based on virtual reality technology. The application aims to create immersive experiences for students by simulating historical scenes in mathematics, using virtual reality technology to provide realistic visual and auditory experiences. Users will be able to immerse themselves in important events, mathematical concepts, and the contributions of mathematicians throughout history. The application has several key features:

1) Create Virtual Reality Scenes: Simulate important events and scenes from the history of mathematics. For example, users can virtually visit the studios of ancient mathematicians, observe the process of solving challenging problems, or participate in historical mathematical conferences and discussions.

2) Interactive Operations in Virtual Environment: Utilize virtual reality technology to allow users to interact within the virtual environment. Users can directly experience the exploration and experimentation of mathematical concepts, such as drawing geometric shapes, solving algebraic equations, and observing mathematical models.

3) Virtual Guided Tours and Commentary: Provide virtual guides and commentary to assist users in exploring important locations and events in the virtual world. This helps users understand the origins of mathematical concepts, the biographies of mathematicians, and their contributions to the development of mathematics.

4) Virtual Laboratory Environment: Create a virtual laboratory where users can conduct experiments and make discoveries related to mathematical history. Users can operate virtual tools, conduct mathematical experiments, observe results, and analyze them.

5) Interactive Learning Activities: Design interactive learning activities related to virtual scenes, such as puzzles, interactive quizzes, and simulated experiments. This allows users to deepen their understanding of concepts and events in mathematical history.

6) Enhance Immersion with VR Technology: Use virtual reality technology to create realistic visual and auditory experiences, enhancing user immersion. Through appropriate visual effects, sound effects, and haptic feedback, users can better engage with and understand the scenes and content of mathematical history.

7) Provide Varied Difficulty Levels: Offer virtual scenes with different levels of difficulty and depth to meet the learning needs and interests of different users. Users can choose the basic mode to understand fundamental concepts in mathematical history or opt for the advanced mode to delve into specific historical events or mathematicians.

In summary, Math VR represents an innovative approach to mathematics education, utilizing virtual reality to provide an engaging and educational experience that transcends traditional learning methods.

3.4 Combine Adventure Games to Experience the Interesting Life of Mathematics

Design of a Role-Playing Game "Math Quest" Grounded in the History of Mathematics.

In the envisioned game, "Math Quest," players assume the role of a young apprentice mathematician summoned by a renowned mathematician in a fantastical world brimming with magic and puzzles. Embarking on an adventurous journey, players traverse diverse eras and locations, delving into the development of mathematical history through the resolution of mathematical problems, puzzles, engaging in combat, and interacting with non-player characters (NPCs). Progression unlocks new levels and content, with interactions involving historical mathematicians and mystical mathematical creatures, unraveling mathematical mysteries, challenging malevolent forces, and ultimately revealing the secrets within the history of mathematics.

1) Character Customization and Mentor Selection: Players can customize the apprentice

mathematician's gender, appearance, and skill traits. Additionally, they can choose a mathematician as a mentor, each offering unique skills and special abilities.

2) Open-World Exploration and Interaction: Players have the freedom to explore diverse in-game locations, engaging in dialogue with NPCs to acquire tasks, hints, and gain insights into mathematical history. Cooperative or competitive interactions with other players are also possible.

3) Mathematical Problem-Solving and Puzzles: Throughout the game, players encounter various mathematical problems and puzzles, ranging from derivations of mathematical formulas to the construction of geometric shapes and solving algebraic equations. Successful problem-solving yields rewards, propelling the storyline.

4) Combat System: The game incorporates a combat system where players engage in battles with hostile mathematical creatures. The turn-based, strategic combat allows players to utilize mathematical concepts and skills for both offensive and defensive maneuvers.

5) Mathematics History Knowledge Base: A knowledge repository within the game provides players with on-demand access to significant events in mathematical history, biographies of mathematicians, and detailed explanations of mathematical concepts.

6) Multimedia Elements: Utilizing multimedia elements such as images, audio, and animations, the game vividly presents crucial events, mathematical works, and the personas of historical mathematicians, enhancing player immersion and learning experiences.

7) Multi-Level Structure: The game comprises multiple levels, each representing an era or event in mathematical history. To progress to the next level, players must complete a series of tasks, challenges, and solve mathematical problems and puzzles. Completion of each level rewards players with new skills and incentives for better tackling subsequent challenges.

8) Experience Points and Skill Development: Players accumulate experience points by completing tasks and solving problems, advancing their character levels, and earning skill points to enhance their abilities. Skill points can be allocated to different mathematical domains, such as algebra, geometry, probability, enabling the adept handling of diverse mathematical challenges.

This conceptual framework integrates immersive gameplay with educational elements, fostering an engaging environment for players to explore the rich tapestry of mathematical history.

4. Summary

The integration of History and Pedagogy of Mathematics (HPM) and Computer Aided Instruction (CAI) can create a math instructional environment that places a heightened emphasis on individual student characteristics and emotional experiences, while leveraging digital technology and personalized support. By combining the theoretical frameworks of HPM and CAI, instructional focus extends beyond the mere concepts and skills of the discipline to encompass students' emotional experiences, attitudes towards the subject, and connections to the historical context of mathematics. This holistic approach aims to enhance students' interest in and profound comprehension of mathematics [11]. The realization of this integration may be facilitated through innovative pedagogical design and the judicious application of digital tools.

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